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IMAGING 2-D OBJECTS HAVING FRACTAL EDGES: CHARACTERISTICS

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ABSTRACT

The image characteristics of a two-dimensional object bounded by fractal-like edges were investigated. The two -dimensional (2-D) object is formed using the star of David as initiator and the von Koch snowflake as generator. The fractal complexity of the object edges is analyzed up to the 10th iteration. The image of the 2-D object is computed by convolving the 2-D object with a sombrero function representing a circular pupil function. Image characteristics, particularly the degradation of details, are analyzed by studying the effects of convolution on the image area and circumference, as well as fractal (Hausdorff-Besicovitch) dimension of the edges of objects formed at various stages of iteration.

PERSPECTIVE

Two general spatial features are of interest when observing a two dimensional object. These are its surface and edges. Surface properties are of particular importance in reflectance, refractive index and contour studies, while an accurate knowledge of edge boundaries is essential in the determination of object sizes, shape and location.

The edge usually lacks the finer features of the true object due to low-pass filtering by the imaging system. These losses of edge details comprise the accuracy with which the object area and location are measured.

The aim of this work was to study how bounded objects with fractal-like edges looked when imaged. Fractals are known to exhibit the property of self-similarity (Feder, 1988) and are thought to be the appropriate geometry to describe objects that occur in nature (Stevens, 1990). More recently, the microstructures with complicated shapes have been studied and experimented on (Langer, 1992). Therefore, an understanding of their imaging properties may lead to new and more effective signal recovery techniques.

according to $Dx = \lambda f/D$ where f is the focal length of the lens and λ is the illumination wavelength.

Figures 4A and 4B show the images of Figure 1 for different diameters of the pupil: D = 32 pixels and D = 256 pixels, respectively. Note that accompanying the decrease in diameter of the circular pupil is the loss of objects details. This smoothing effect is a consequence of diffraction limited imaging. The images in Figure 4 were generated by taking the inverse Fourier transform of the resulting products between the pupil and the Fourier transform of the object.

Figures 5A and 5B illustrate how the image area and circumference are affected by the imaging process at various diameters of the pupil. Note that the image area is multivalued with respect to D, but the circumference rises monotonically with D. Both graphs are normalized with respect to values associated with the object.

Figure 6 presents the effect of pupil diameter on the fractal dimension of the image. The fractal dimension of the image is lower than that of the object. This decrease is expected as the convolving effect of the pupil function with finite diameter leads to the smoothing of edge details. Note further, that the effect of the pupil diameter on the fractal dimension is nonlinear within range of diameter values considered. The fractal dimension rises monotonically with D.

CONCLUSION

This study aimed to find correlations between the fractal dimension of the image and the perceptible effects of area and circumferential changes as the object with known fractal dimension is imaged using a circular pupil of known diameter. Experimental results show that object smoothing, which is a consequence of diffraction-limited imaging, affects distinctively the object area and circumference. The image circumference increases monotonically as the diameter of the convolving pupil is increased. On the other hand, the image area before finally increasing, exhibits an inflection point around D = 48 pixels. As the D is increased, it is expected that the image area will asymptotically approach the value of the object area.

Results also indicate that fractal dimension is affected nonlinearly by imaging. There is a monotonic dependence of the fractal dimension on pupil diameter.

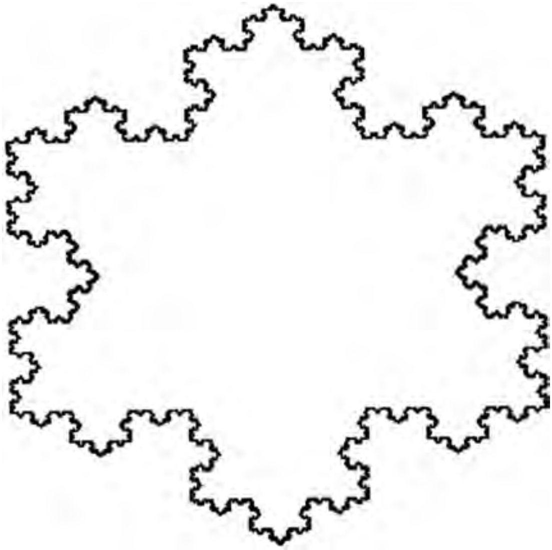

Figure 1. **Edge profile of the Star of David initiator after the 6th iteration of the von Koch snowflake generator**

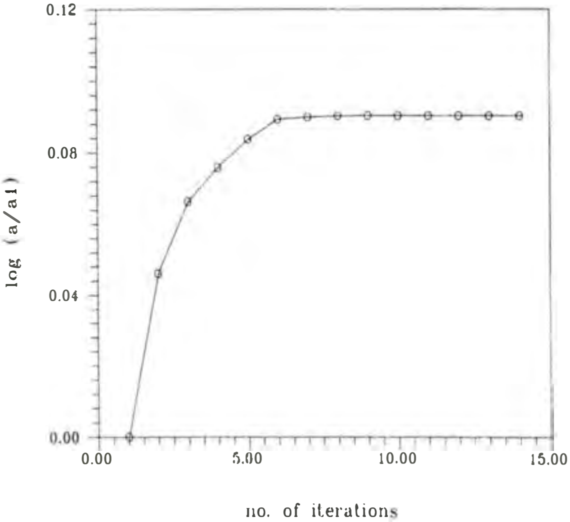

Figure 2(a). **Dependence of object area with the order of iteration**

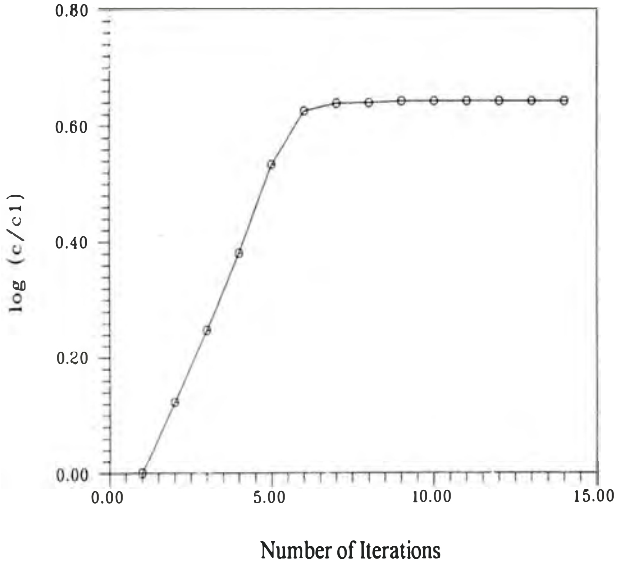

Figure 2(b). Dependence of object circumference with the order of iteration

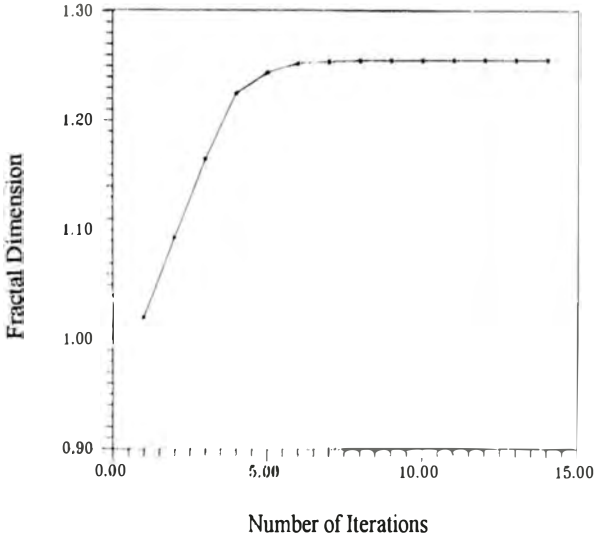

Figure 3. Relationship between fractal dimension and order of iteration

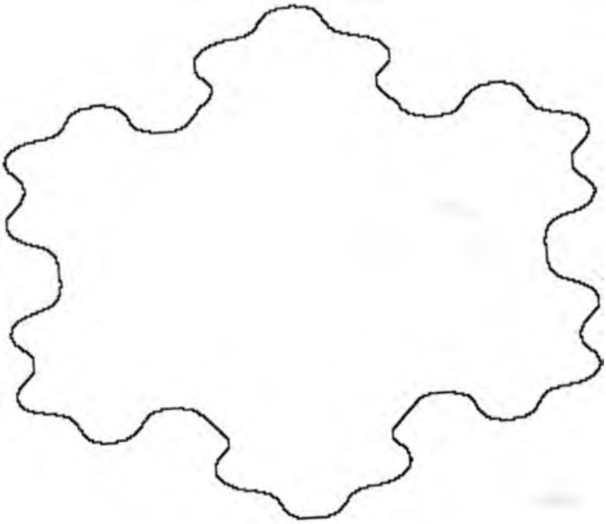

Figure 4(a). Effects of imaging using circular pupil on the object (6th iteration); pupil diameter = 32 pixels. Object used for imaging is that shown in Figure 1.

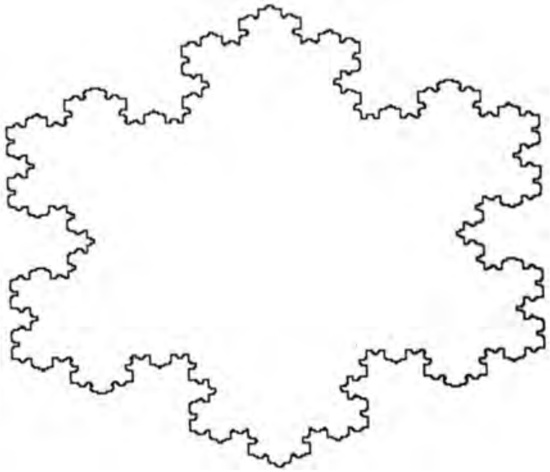

Figure 4(b). Effects of imaging using circular pupil on the object (6th iteration); pupil diameter = 256 pixels. Object used for imaging is that shown in Figure 1.

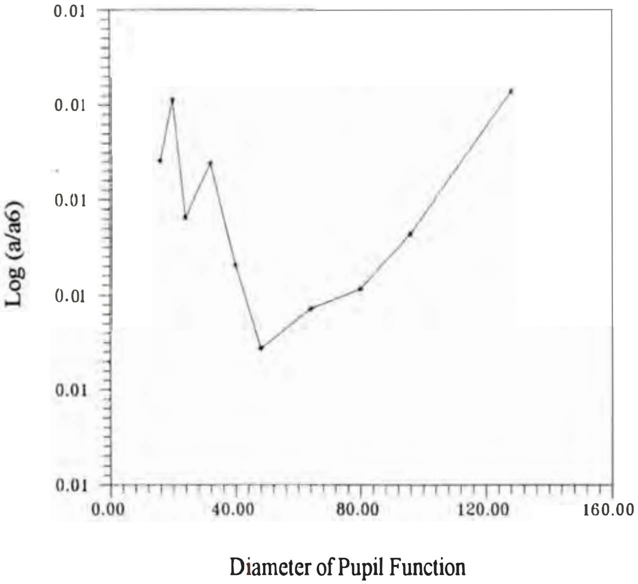

Figure 5(a). Dependence of image area with pupil diameter

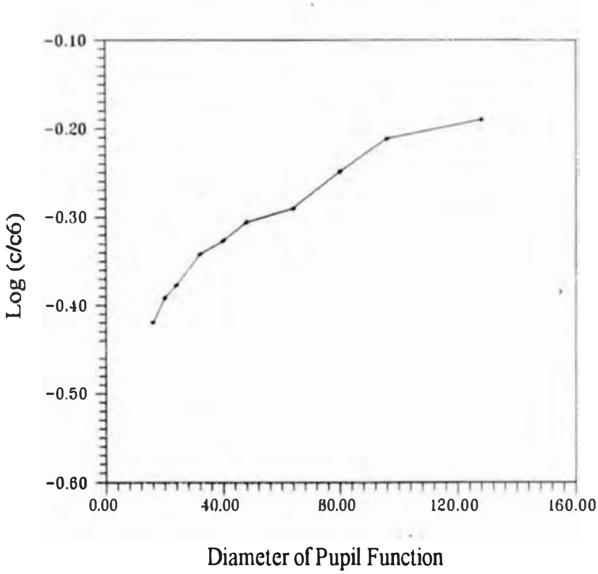

Figure 5(b). Dependence of image circumference with pupil diameter

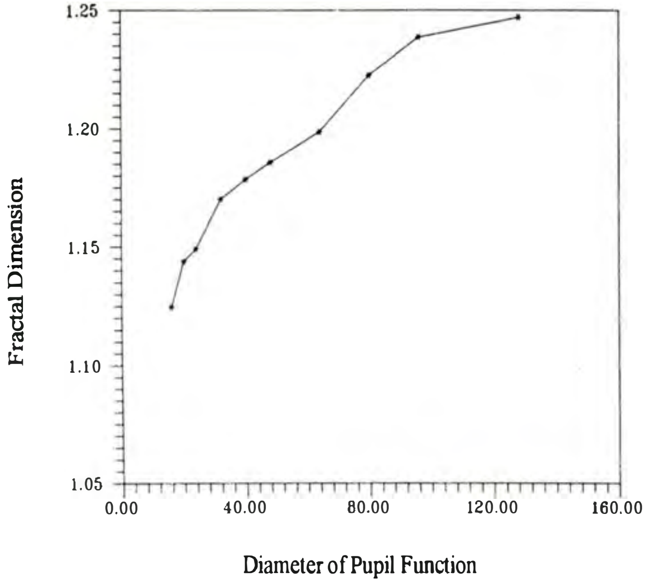

Figure 6. **Relationship between image fractal dimension and pupil diameter**

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